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ABSTRACT

The numerical evaluation of the theoretical variance/mean ratio of the neutron population in the subcritical condition of the AFRRI-TRIGA reactor is given for (1) the case of no delayed neutron groups; (2) the case of one equivalent delayed neutron group; and (3) the case of six delayed neutron groups. The Courant-Wallace equations were solved to obtain the theoretical variance/mean ratio using the mean prompt neutron lifetime of 39×10^{-6} seconds in the AFRRI-TRIGA reactor.

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I. INTRODUCTION

The mean number of neutrons and delayed neutron-forming precursors in a reactor may be estimated by deterministic equations but the actual number at time t will be different. The instantaneous number of neutrons and precursors, both possessing a discrete rather than a continuous probability distribution function, will fluctuate at random about the mean. A statistical parameter which may be used to measure the fluctuations is the variance/mean ratio. References 2 and 6, through the use of the probability balance and two probability generating functions, derive equations describing the stochastic processes within the reactor's core in terms of the first and second moments of the neutron/precursor probability distributions. These equations may be solved for the variance/mean ratio. It is the purpose of this report to numerically evaluate the theoretical variance/mean ratio of the neutron population for three cases using the fission data for ^{235}U from references 3 and 5 and the mean prompt neutron lifetime in the AFRRI-TRIGA reactor which is approximately 39×10^{-6} seconds.[1] The symbolism of reference 6 will be followed. Case 1 is when it is assumed that there are no delayed neutron groups. Case 2 is when there is one equivalent delayed neutron group. Case 3 is for six delayed neutron groups.

II. GLOSSARY OF BASIC SYMBOLS

k = $k' + \sum_i c_i$ = the mean number of neutrons formed (both instantaneous and delayed) per neutron lost in the reactor through absorption or escape.

k' = the mean number of neutrons formed instantaneously per neutron lost in the reactor through absorption or escape.

c = $\sum_i c_i$.

c_i = the mean number of radioactive neutron-forming precursors of type i formed per neutron lost in the reactor through absorption or escape.

ϕ_{NN} = variance of $N = \overline{N^2} - \overline{N}^2$.

ϕ_{Nj} = $\overline{NC_j} - \overline{N}\ \overline{C_j}$.

ϕ_{ij} = $\overline{C_i C_j} - \overline{C_i}\ \overline{C_j}$.

C_i = the total number of precursors of type i in the reactor.

ϵ_i = $\lambda_i \tau$.

λ_i = decay constant of precursor type i.

τ = mean prompt lifetime of a neutron in the reactor.

f = $f(x, y_1, y_2, \ldots)$ = f-type probability generating function

$$= \sum_n \sum_{m_1} \sum_{m_2} \ldots x^n y_1^{m_1} y_2^{m_2} \ldots p(n, m_1, m_2, \ldots).$$

$p(n, m_1, m_2, \ldots)$ = the probability that the absorption or escape of one neutron in the reactor will lead to the formation of n instantaneous neutrons and m_i precursors of type i.

n = number of neutrons formed instantaneously per neutron lost in the reactor through absorption or escape.

m_i = number of precursors of type i formed per neutron lost in the reactor through absorption or escape.

it is to be noted that $k = \overline{n} + \sum_i \overline{m_i}$ since $k' = \overline{n}$ and $c = \sum_i c_i = \overline{m} = \sum_i \overline{m_i}$.

F = $F(x, y_1, y_2, \ldots; t)$ = F-type probability generating function

$$= \sum_N \sum_{C_1} \sum_{C_2} \ldots x^N y_1^{C_1} y_2^{C_2} \ldots P(N, C_1, C_2, \ldots; t).$$

$P(N, C_1, C_2, \ldots; t)$ = the probability that N neutrons, C_1 precursors of type 1, C_2 precursors of type 2, etc., are present in the reactor at time t.

t = time.

N = the total number of neutrons in the reactor.

subscripts on f and F - the subscripts on f and F indicate partial differentiation, i.e.,

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \text{ and}$$

$$f_{ix} = \frac{\partial}{\partial y_i}\left(\frac{\partial f}{\partial x}\right) \text{ and}$$

$$f_{ij} = \frac{\partial}{\partial y_i}\left(\frac{\partial f}{\partial y_j}\right)$$

$\langle\ \rangle$ - the angle brackets indicate evaluation of the enclosed term at the point $x = y_i = 1$.

$\overline{N}$ - the bar over a symbol indicates the mean value, thus, $\overline{N}$ = the mean value of N.

δ_{ij} = Kronecker delta

$\delta_{ij} = 0$ when $i \neq j$

$\delta_{ij} = 1$ when $i = j$.

ξ = the probability of achieving a fission event per neutron lost through absorption or escape.

ν = the number of neutrons formed per fission event.

β = the mean number of delayed precursors formed per fission neutron produced.

β_i = the mean number of delayed precursors of type i formed per fission neutron produced.

prime symbol - the prime symbol, except for k', is a shortened notation to indicate division by $\overline{N}$, i.e., $\phi'_{NN} = \frac{\phi_{NN}}{\overline{N}}$.

k_2 = mean square of the number of neutrons formed per neutron lost through absorption or escape.

A = 1-k+c.

T_i = half-period of radioactive precursor of type i.

α_i = percent yield of $\nu\beta$ precursors of type i.

III. GENERAL EQUATIONS

The equations in references 2 and 6, describing the fluctuations of the neutron/precursor probability distributions, are

$$(1-k+c)\,\phi_{NN} - \sum_{i=1}^{6} \epsilon_i\,\phi_{Ni} = \left[\frac{1}{2}\langle f_{xx}\rangle + (1-k+c)\right]\overline{N} \tag{1}$$

which results in one equation as we let i = 1,2,3,4,5, and 6, and

$$-c_i\,\phi_{NN} + (1-k+c+\epsilon_i)\,\phi_{Ni} - \sum_{j=1}^{6} \epsilon_j\,\phi_{ij} = \left[\langle f_{ix}\rangle - 2c_i\right]\overline{N} \tag{2}$$

which results in six equations for the six values of i, and

$$-c_i\,\phi_{Nj} - c_j\,\phi_{Ni} + (\epsilon_j + \epsilon_i)\,\phi_{ij} = \left[\langle f_{ij}\rangle + \delta_{ij}\,2c_i\right]\overline{N} \tag{3}$$

which results in 21 equations when i = 1,2,3,4,5, and 6, and when j = 1,2,3,4,5, and

6 since the mixed partial derivatives of ϕ, namely, $\phi_{i_A j_A} = \phi_{j_B i_B}$, are equal when $i_A = i_B$ and when $j_A = j_B$.

IV. CASE 1: NO DELAYED NEUTRONS

The assumption of no delayed neutron groups means we assume that $c = c_i = 0$, $m_i = 0$, and that $k = k'$. Equations (1), (2), and (3) reduce to

$$(1-k)\,\phi_{NN} = (\tfrac{1}{2} f_{xx} + 1-k)\ \overline{N}\ . \tag{4}$$

With $m_i = 0$, the f-type probability generating function becomes

$$f(x) = \sum_n x^n\ p(n) \tag{5}$$

and

$$f(x)_{xx} = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \sum_n n(n-1)x^{n-2}\ p(n) \tag{6}$$

and

$$\langle f_{xx}\rangle = \left[\sum_n n(n-1)\ x^{n-2}\ p(n)\right]_{x=1} \tag{7}$$

$$= \sum_n n(n-1)\ p(n) \tag{8}$$

$$= \sum_n n^2\ p(n) - \sum_n n\ p(n) \tag{9}$$

$$= \overline{n^2} - \overline{n}\ . \tag{10}$$

Using (10), equation (4) rearranges to

$$\frac{\phi_{NN}}{\overline{N}} = 1 + \frac{\overline{n^2} - \overline{n}}{2(1-k)}\ . \tag{11}$$

By definition, k, the effective multiplication factor of the reactor system, is the mean number of neutrons formed per neutron lost in the system through escape or absorption. Thus

$$k = \overline{\xi \nu} \tag{12}$$

where ξ = the probability of achieving a fission event per neutron lost through absorption or escape and where ν = the number of neutrons formed per fission event.

Assuming that ξ and ν are two independent sets (events),

$$k = \overline{\xi \nu} = \overline{\xi}\ \overline{\nu} = \xi \overline{\nu} \tag{13}$$

since by definition $\overline{\xi} = \xi$.

Since $k = k'$, therefore

$$k = \xi \overline{\nu} = \overline{n} \ . \tag{14}$$

Thus

$$\overline{n^2} = \overline{(\xi \nu)^2} = \xi^2 \ \overline{\nu^2} \ . \tag{15}$$

Substitute (15) into (11), and obtain

$$\frac{\phi_{NN}}{\overline{N}} = 1 + \frac{\xi^2 \ \overline{\nu^2} - k}{2(1-k)} \quad . \tag{16}$$

By selecting values for the dependent variable k we may obtain the variance/mean ratio of the neutron population for various conditions of subcriticality for the no delayed neutron case using equation (16). Table II (page 17) and Figures 1 and 2 (page 18) summarize the numerical results.

V. CASE 2: ONE DELAYED NEUTRON GROUP

To obtain equations for one equivalent delayed neutron group we let $c_i = c$. Equations (1), (2), and (3) become

$$(1-k+c)\,\phi_{NN} - \epsilon\,\phi_{NC} = (\tfrac{1}{2}\langle f_{xx}\rangle + 1-k+c)\,\overline{N} \tag{17}$$

$$-c\,\phi_{NN} + (1-k+c+\epsilon)\,\phi_{NC} - \epsilon\,\phi_{CC} = (\langle f_{xy}\rangle - 2c)\,\overline{N} \tag{18}$$

$$-c\,\phi_{NC} + \epsilon\,\phi_{CC} = \left(\frac{1}{2}\langle f_{yy}\rangle + c\right)\overline{N} \tag{19}$$

where

$$\phi_{NC} = \overline{NC} - \overline{N}\;\overline{C} \tag{20}$$

and where

$$\phi_{CC} = \overline{C^2} - \overline{C}^2 \;. \tag{21}$$

Add (17), (18), and (19) and obtain

$$(1-k)\;(\phi_{NN} + \phi_{NC}) = \left(\frac{1}{2}\langle f_{xx}\rangle + \langle f_{xy}\rangle + \frac{1}{2}\langle f_{yy}\rangle + 1 - k\right)\overline{N}\;. \tag{22}$$

Since

$$\langle f_{xx}\rangle = \sum_{n,m} n(n-1)\;p(n,m) \tag{23}$$

and

$$\langle f_{xy}\rangle = \sum_{n,m} nm\;p(n,m) \tag{24}$$

and

$$\langle f_{yy}\rangle = \sum_{n,m} m(m-1)\;p(n,m) \tag{25}$$

and

$$(k_2-k) = \sum_{n,m} \left[(n+m)^2 - (n+m)\right] p(n,m) \tag{26}$$

it is seen that

$$\frac{1}{2}\,(k_2-k) = \frac{1}{2}\langle f_{xx}\rangle + \langle f_{xy}\rangle + \frac{1}{2}\langle f_{yy}\rangle \;. \tag{27}$$

Substitute (27) into (22) and obtain

$$(1-k)\;(\phi_{NN} + \phi_{NC}) = \left[\frac{1}{2}\,(k_2-k) + (1-k)\right]\overline{N}\;. \tag{28}$$

Solve (17), (19), and (28) for $\phi_{NN}/\overline{N}$ by Cramer's rule (see reference 6 for details) and obtain

$$\frac{\phi_{NN}}{\overline{N}} = 1 + \frac{(k_2-k)(1-k+\epsilon)}{2(1-k)(1-k+c+\epsilon)} - \frac{\frac{1}{2}c'}{1-k+c+\epsilon} \tag{29}$$

where

$$c' = \left[\, 2f_{xy} + f_{yy} \,\right]_{x=y=1} \tag{30}$$

$$= 2\,\overline{nm} + \overline{m^2} - \overline{m}\,. \tag{31}$$

By definition, c is the mean number of precursors formed per neutron lost through absorption or escape. Thus

$$c = k\beta \tag{32}$$

where β = the mean number of delayed precursors formed per fission neutron produced.

For the AFRRI-TRIGA reactor

$$\epsilon = \lambda\tau \tag{33}$$

$$\cong 0.077 \times 39 \times 10^{-6} \tag{34}$$

$$\cong 3.0 \times 10^{-6} \tag{35}$$

where 0.077 sec^{-1} is the equivalent one-group decay constant computed from the 6-group thermal fission data in reference 5 by the following expression

$$\lambda_{\substack{\text{equivalent}\\ \text{1-group}}} = \frac{\beta}{\sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}}\,. \tag{36}$$

By definition m is the number of delayed neutrons formed per neutron lost. Thus

$$c = \overline{m} = \overline{\xi\nu\beta}\,. \tag{37}$$

It is assumed that ξ, ν, and β are statistically independent. Hence

$$\overline{m} = \overline{\xi\nu\beta} = \overline{\xi}\ \overline{\nu}\ \overline{\beta} = \xi\ \overline{\nu}\ \beta \tag{38}$$

and

$$\overline{m^2} = \overline{(\xi\nu\beta)^2} = \overline{\xi^2}\ \overline{\nu^2}\ \overline{\beta^2} = \xi^2\ \overline{\nu^2}\ \beta^2\,. \tag{39}$$

By the same logic

$$\overline{nm} = \overline{(\xi\nu\ (1-\beta) \times \xi\nu\beta\ )} \tag{40}$$

$$= \xi^2\ \overline{\nu^2}\ \beta\ -\ \xi^2\ \overline{\nu^2}\ \beta^2\ . \tag{41}$$

Substitute (38), (39), and (41) into (31) and obtain

$$c' \simeq 2\ \xi^2\ \overline{\nu^2}\ \beta\ -\ \xi\ \overline{\nu}\ \beta \tag{42}$$

where the second-order term involving β^2 is neglected.

It is realized that the assumption of statistical independence between ν and β is an approximation and that the result of this assumption is to state in the averaging process that the mean of the product of ν and β is equal to the product of the separate means when in actuality the mean of $\nu\beta$ should involve sums of terms like $\nu\beta$ times an unconditional probability and a conditional probability. Equation (38) states, in essence, that we are considering ξ and β as constants and are combining the fluctuations of the neutron and precursor formations into the ν value. Overall, the value of the last term in (29), the only term involving c' and hence (38), is, in any case, small, and thus to a first order approximation the assumption of statistical independence between ν and β introduces only a negligible error which is of a magnitude less than the standard deviation of the final ratio obtained by propagating the precision indexes of the experimental data for the various other parameters in (29).

Substitute (13), (37), and (42) into (29) and obtain

$$\frac{\phi_{NN}}{\overline{N}} \simeq 1 + \frac{\xi^2\ \overline{\nu^2}\ (1-2\beta)\ -\ k}{2\ (\ (1-k)\ +\ k\beta\ )} \tag{43}$$

where it is to be noted in arriving at the solution that the constant ϵ is of negligible importance.

Table II (page 17) and Figures 1 and 2 (page 18) summarize the $\phi_{NN}/\overline{N}$ ratios for various values of k.

VI. CASE 3: SIX DELAYED NEUTRON GROUPS

Twenty-eight simultaneous equations resulting from the expansions of (1), (2), and (3) must be solved to obtain the $\phi_{NN}/\overline{N}$ ratio in the case of six delayed neutron groups. These 28 equations are

$$A\cdot\phi'_{NN} - \epsilon_1\phi'_{N1} - \epsilon_2\phi'_{N2} - \epsilon_3\phi'_{N3} - \epsilon_4\phi'_{N4} - \epsilon_5\phi'_{N5} - \epsilon_6\phi'_{N6} = \left[\frac{1}{2}\langle f_{xx}\rangle + A\right] \quad (44)$$

$$-c_1\phi'_{NN} + (A+\epsilon_1)\phi'_{N1} - \epsilon_1\phi'_{11} - \epsilon_2\phi'_{12} - \epsilon_3\phi'_{13} - \epsilon_4\phi'_{14} - \epsilon_5\phi'_{15} - \epsilon_6\phi'_{16} = \langle f_{1x}\rangle - 2c_1 \quad (45)$$

$$-c_2\phi'_{NN} + (A+\epsilon_2)\phi'_{N2} - \epsilon_1\phi'_{21} - \epsilon_2\phi'_{22} - \epsilon_3\phi'_{23} - \epsilon_4\phi'_{24} - \epsilon_5\phi'_{25} - \epsilon_6\phi'_{26} = \langle f_{2x}\rangle - 2c_2 \quad (46)$$

$$-c_3\phi'_{NN} + (A+\epsilon_3)\phi'_{N3} - \epsilon_1\phi'_{31} - \epsilon_2\phi'_{32} - \epsilon_3\phi'_{33} - \epsilon_4\phi'_{34} - \epsilon_5\phi'_{35} - \epsilon_6\phi'_{36} = \langle f_{3x}\rangle - 2c_3 \quad (47)$$

$$-c_4\phi'_{NN} + (A+\epsilon_4)\phi'_{N4} - \epsilon_1\phi'_{41} - \epsilon_2\phi'_{42} - \epsilon_3\phi'_{43} - \epsilon_4\phi'_{44} - \epsilon_5\phi'_{45} - \epsilon_6\phi'_{46} = \langle f_{4x}\rangle - 2c_4 \quad (48)$$

$$-c_5\phi'_{NN} + (A+\epsilon_5)\phi'_{N5} - \epsilon_1\phi'_{51} - \epsilon_2\phi'_{52} - \epsilon_3\phi'_{53} - \epsilon_4\phi'_{54} - \epsilon_5\phi'_{55} - \epsilon_6\phi'_{56} = \langle f_{5x}\rangle - 2c_5 \quad (49)$$

$$-c_6\phi'_{NN} + (A+\epsilon_6)\phi'_{N6} - \epsilon_1\phi'_{61} - \epsilon_2\phi'_{62} - \epsilon_3\phi'_{63} - \epsilon_4\phi'_{64} - \epsilon_5\phi'_{65} - \epsilon_6\phi'_{66} = \langle f_{6x}\rangle - 2c_6 \quad (50)$$

$$-2c_1\phi'_{N1} + 2\epsilon_1\phi'_{11} = \langle f_{11}\rangle + 2c_1 \quad (51)$$

$$-c_1\phi'_{N2} - c_2\phi'_{N1} + (\epsilon_2 + \epsilon_1)\phi'_{12} = \langle f_{12}\rangle \quad (52)$$

$$-c_1\phi'_{N3} - c_3\phi'_{N1} + (\epsilon_3 + \epsilon_1)\phi'_{13} = \langle f_{13}\rangle \quad (53)$$

$$-c_1\phi'_{N4} - c_4\phi'_{N1} + (\epsilon_4 + \epsilon_1)\phi'_{14} = \langle f_{14}\rangle \quad (54)$$

$$-c_1\phi'_{N5} - c_5\phi'_{N1} + (\epsilon_5 + \epsilon_1)\phi'_{15} = \langle f_{15}\rangle \quad (55)$$

$$-c_1\phi'_{N6} - c_6\phi'_{N1} + (\epsilon_6 + \epsilon_1)\phi'_{16} = \langle f_{16}\rangle \quad (56)$$

$$-2c_2\phi'_{N2} + 2\epsilon_2\phi'_{22} = \langle f_{22}\rangle + 2c_2 \tag{57}$$

$$-c_2\phi'_{N3} - c_3\phi'_{N2} + (\epsilon_3 + \epsilon_2)\phi'_{23} = \langle f_{23}\rangle \tag{58}$$

$$-c_2\phi'_{N4} - c_4\phi'_{N2} + (\epsilon_4 + \epsilon_2)\phi'_{24} = \langle f_{24}\rangle \tag{59}$$

$$-c_2\phi'_{N5} - c_5\phi'_{N2} + (\epsilon_5 + \epsilon_2)\phi'_{25} = \langle f_{25}\rangle \tag{60}$$

$$-c_2\phi'_{N6} - c_6\phi'_{N2} + (\epsilon_6 + \epsilon_2)\phi'_{26} = \langle f_{26}\rangle \tag{61}$$

$$-2c_3\phi'_{N3} + 2\epsilon_3\phi'_{33} = \langle f_{33}\rangle + 2c_3 \tag{62}$$

$$-c_3\phi'_{N4} - c_4\phi'_{N3} + (\epsilon_4 + \epsilon_3)\phi'_{34} = \langle f_{34}\rangle \tag{63}$$

$$-c_3\phi'_{N5} - c_5\phi'_{N3} + (\epsilon_5 + \epsilon_3)\phi'_{35} = \langle f_{35}\rangle \tag{64}$$

$$-c_3\phi'_{N6} - c_6\phi'_{N3} + (\epsilon_6 + \epsilon_3)\phi'_{36} = \langle f_{36}\rangle \tag{65}$$

$$-2c_4\phi'_{N4} + 2\epsilon_4\phi'_{44} = \langle f_{44}\rangle + 2c_4 \tag{66}$$

$$-c_4\phi'_{N5} - c_5\phi'_{N4} + (\epsilon_5 + \epsilon_4)\phi'_{45} = \langle f_{45}\rangle \tag{67}$$

$$-c_4\phi'_{N6} - c_6\phi'_{N4} + (\epsilon_6 + \epsilon_4)\phi'_{46} = \langle f_{46}\rangle \tag{68}$$

$$-2c_5\phi'_{N5} + 2\epsilon_5\phi'_{55} = \langle f_{55}\rangle + 2c_5 \tag{69}$$

$$-c_5\phi'_{N6} - c_6\phi'_{N5} + (\epsilon_6 + \epsilon_5)\phi'_{56} = \langle f_{56}\rangle \tag{70}$$

$$-2c_6\phi'_{N6} + 2\epsilon_6\phi'_{66} = \langle f_{66}\rangle + 2c_6 \tag{71}$$

where
$$A = 1-k+c \tag{72}$$

and where the prime mark over the ϕ indicates division by $\overline{N}$, i.e.,

$$\phi'_{NN} = \frac{\phi_{NN}}{\overline{N}} \ . \tag{73}$$

The constants used in solving (44) through (71) were obtained from ^{235}U neutron induced fission data in reference 3 and from the thermal fission delayed neutron data for ^{235}U in reference 5 and are listed below.

$$\bar{\nu} = 2.47 \pm 0.03 \text{ (± standard deviation)} \tag{74}$$

$$\overline{\nu^2} = 7.32 \pm 0.15 \tag{75}$$

$$\beta = 0.0064 \pm 0.0003 \; . \tag{76}$$

Substitute (32) into (72) and obtain

$$A = 1-k+c \tag{77}$$

$$= 1.0 - 0.9936 \, k \; . \tag{78}$$

By definition

$$\epsilon_i = \lambda_i \tau \tag{79}$$

since

$$\lambda_i = \frac{\ln 2}{T_i} \tag{80}$$

where T_i is the half-period as described on page 473 of reference 4.

Substitute (80) into (79) and obtain

$$\epsilon_i = \left(\frac{\ln 2}{T_i} \right) \tau \; . \tag{81}$$

The six values of T_i from reference 5 are 55.72, 22.72, 6.22, 2.30, 0.610, and 0.230 sec for delayed neutron groups i = 1 through i = 6 respectively. Inserting these values into (81) we obtain

$$\epsilon_1 = 0.48515 \times 10^{-6} \tag{82}$$

$$\epsilon_2 = 1.18982 \times 10^{-6} \tag{83}$$

$$\epsilon_3 = 4.34610 \times 10^{-6} \tag{84}$$

$$\epsilon_4 = 11.75336 \times 10^{-6} \tag{85}$$

$$\epsilon_5 = 44.31597 \times 10^{-6} \tag{86}$$

$$\epsilon_6 = 117.53365 \times 10^{-6} \; . \tag{87}$$

By definition c_i is the mean number of precursors of type i formed per neutron lost through absorption or escape. Thus by analogy to (32)

$$c_i = k\beta_i \tag{88}$$

where β_i is the mean number of delayed precursors of type i formed per fission neutron produced.

Thus (88) may be computed by the expression

$$c_i = \frac{k\alpha_i}{2.47 \times 100} = \frac{k\alpha_i}{247} \tag{89}$$

where α_i is the percent yield of $\nu\beta$ of precursors of type i.

Substitute the six values of α_i from reference 5, namely, 0.052, 0.346, 0.310, 0.624, 0.182, and 0.066, into (89) and obtain

$$c_1 = 2.112 \times 10^{-4} \times k \tag{90}$$

$$c_2 = 14.016 \times 10^{-4} \times k \tag{91}$$

$$c_3 = 12.544 \times 10^{-4} \times k \tag{92}$$

$$c_4 = 25.280 \times 10^{-4} \times k \tag{93}$$

$$c_5 = 7.360 \times 10^{-4} \times k \tag{94}$$

$$c_6 = 2.688 \times 10^{-4} \times k\ . \tag{95}$$

By (23) and (40)

$$\langle f_{xx} \rangle = \overline{n(n-1)} = \overline{n^2} - \overline{n} \tag{96}$$

$$= \xi^2 \overline{\nu^2} (1-\beta)^2 - \xi \overline{\nu} (1-\beta)\ . \tag{97}$$

By definition and by analogy to (40)

$$\langle f_{ix} \rangle = \overline{m_i n}$$

$$= \overline{[\xi \nu \beta_i]\ [\xi \nu (1-\beta)]} \tag{98}$$

$$= \xi^2 \overline{\nu^2} \beta_i (1-\beta) \ . \tag{99}$$

Similarly when $i \neq j$

$$\langle f_{ij} \rangle = \overline{m_i m_j} = \overline{(\xi \nu \beta_i)\ (\xi \nu \beta_j)} \tag{100}$$

$$= \xi^2 \overline{\nu^2} \beta_i \beta_j \tag{101}$$

and when $i = j$

$$\langle f_{ij} \rangle = \langle f_{ii} \rangle = \overline{m_i (m_i - 1)} \tag{102}$$

$$= \overline{[\xi \nu \beta_i]\ [\xi \nu \beta_i - 1]} \tag{103}$$

$$= \xi^2 \overline{\nu^2} \beta_i^{\ 2} - \xi \bar{\nu} \beta_i \ . \tag{104}$$

The assumption of statistical independence between ν and β is used in the evaluation of $\langle f_{xx} \rangle$, $\langle f_{ix} \rangle$, and $\langle f_{ij} \rangle$.

Substitute (13), (74), (75), (76), and (89) into (97), (99), (101), and (104) and obtain

$$\frac{1}{2} \langle f_{xx} \rangle = 0.59226\ k^2 - 0.49680\ k \tag{105}$$

$$\langle f_{1x} \rangle = k^2 \times 2.51780 \times 10^{-4} \tag{106}$$

$$\langle f_{2x} \rangle = k^2 \times 16.70903 \times 10^{-4} \tag{107}$$

$$\langle f_{3x} \rangle = k^2 \times 14.95420 \times 10^{-4} \tag{108}$$

$$\langle f_{4x} \rangle = k^2 \times 30.13730 \times 10^{-4} \tag{109}$$

$$\langle f_{5x} \rangle = k^2 \times 8.77415 \times 10^{-4} \tag{110}$$

$$\langle f_{6x} \rangle = k^2 \times 3.20447 \times 10^{-4} \tag{111}$$

$$\langle f_{11} \rangle = k^2 \times 10^{-8} \times 5.35185 - 2.112 \times 10^{-4}\ k \tag{112}$$

$$\langle f_{12} \rangle = k^2 \times 10^{-8} \times 35.51682 \tag{113}$$

$$\langle f_{13} \rangle = k^2 \times 10^{-8} \times 31.78675 \tag{114}$$

$$\langle f_{14} \rangle = k^2 \times 10^{-8} \times 64.06003 \tag{115}$$

$$\langle f_{15} \rangle = k^2 \times 10^{-8} \times 18.65039 \tag{116}$$

$$\langle f_{16} \rangle = k^2 \times 10^{-8} \times \ 6.81145 \tag{117}$$

$$\langle f_{22} \rangle = k^2 \times 10^{-8} \times 235.70258 - 14.016 \times 10^{-4}\ k \tag{118}$$

$$\langle f_{23} \rangle = k^2 \times 10^{-8} \times 210.94843 \tag{119}$$

$$\langle f_{24} \rangle = k^2 \times 10^{-8} \times 425.12567 \tag{120}$$

$$\langle f_{25} \rangle = k^2 \times 10^{-8} \times 123.77076 \tag{121}$$

$$\langle f_{26} \rangle = k^2 \times 10^{-8} \times \ 45.20324 \tag{122}$$

$$\langle f_{33} \rangle = k^2 \times 10^{-8} \times 188.79297 - 12.544 \times 10^{-4}\ k \tag{123}$$

$$\langle f_{34} \rangle = k^2 \times 10^{-8} \times 380.47765 \tag{124}$$

$$\langle f_{35} \rangle = k^2 \times 10^{-8} \times 110.77197 \tag{125}$$

$$\langle f_{36} \rangle = k^2 \times 10^{-8} \times \ 40.45585 \tag{126}$$

$$\langle f_{44} \rangle = k^2 \times 10^{-8} \times 766.77905 - 25.280 \times 10^{-4}\ k \tag{127}$$

$$\langle f_{45} \rangle = k^2 \times 10^{-8} \times 223.23210 \tag{128}$$

$$\langle f_{46} \rangle = k^2 \times 10^{-8} \times \ 81.53093 \tag{129}$$

$$\langle f_{55} \rangle = k^2 \times 10^{-8} \times \ 64.99377 - 7.360 \times 10^{-4}\ k \tag{130}$$

$$\langle f_{56} \rangle = k^2 \times 10^{-8} \times \ 23.73685 \tag{131}$$

$$\langle f_{66} \rangle = k^2 \times 10^{-8} \times \ 8.66911 - 2.688 \times 10^{-4}\ k\ . \tag{132}$$

The 28 equations, (44) through (71), were solved by hand on a Monroe desk calculator by Robert W. Rockwell. The inverse matrix involved contained 683 zeros. The Gauss-Seidel method was used to invert the matrix.

Table II and Figures 1 and 2 summarize the six-group $\phi_{NN}/\overline{N}$ ratios for various values of k.

VII. PROPAGATION OF PRECISION INDEXES

The propagation of precision indexes was accomplished for three values of k for Case 2 in order to obtain a feeling for the magnitude of the uncertainty in the variance/mean ratio. Following the procedures given on pages 205-208 of reference 7 and using the constants of (74), (75) and (76), the following results were obtained and are tabulated below in Table I and shown on Figure 2.

Table I

Theoretical Variance/Mean Ratios ± Standard Deviations of the Neutron Population for Three Values of k for Case 2 (One Delayed Neutron Group)

k	variance/mean ratio ± standard deviation
0.93	1.62 ± 0.18
0.97	2.99 ± 0.41
0.995	8.81 ± 1.38

VIII. NUMERICAL RESULTS

Table II and Figures 1 and 2 tabulate and plot the theoretical variance/mean ratios of the neutron population in the subcritical condition of the AFRRI-TRIGA reactor.

Table II

Theoretical Variance/Mean Ratios of the Neutron Population in the Subcritical Condition of the AFRRI-TRIGA Reactor

k	variance/mean ratio		
	Case 1*	Case 2**	Case 3***
0	1.000	1.000	--
0.10	0.951	0.951	--
0.20	0.905	0.905	--
0.30	0.863	0.862	--
0.40	0.827	0.825	--
0.50	0.800	0.797	--
0.60	0.790	0.785	--
0.70	0.813	0.804	--
0.80	0.920	0.898	--
0.85	1.056	1.019	1.037
0.86	1.098	1.055	1.074
0.87	1.147	1.098	1.118
0.88	1.205	1.148	1.171
0.89	1.274	1.208	1.233
0.90	1.359	1.281	1.307
0.91	1.464	1.370	1.400
0.92	1.597	1.480	1.515
0.93	1.769	1.622	1.661
0.94	2.001	1.807	1.853
0.95	2.328	2.061	2.116
0.96	2.822	2.426	2.493
0.97	3.649	2.995	3.081
0.98	5.308	3.999	4.120
0.99	10.297	6.231	6.429
0.995	20.285	8.814	9.106
0.9995	200.1	14.324	14.918
0.99995	1998	15.295	16.95
0.999995	19982	15.400	17.20

* Case 1 (no delayed neutron groups) was computed from equation (16).
** Case 2 (one delayed neutron group) was computed from equation (43).
*** Case 3 (six delayed neutron groups) was computed from equations (44) through (71).

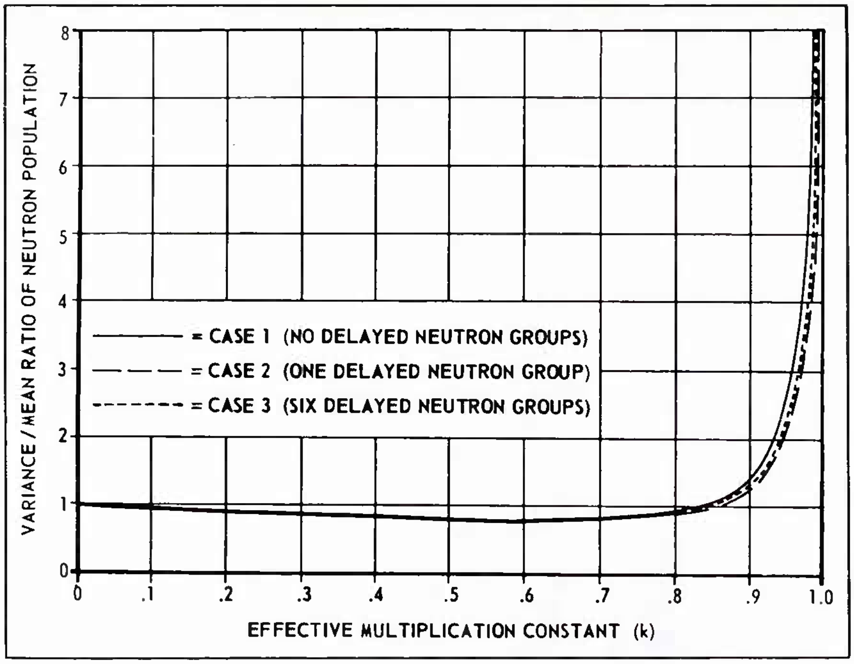

Figure 1. Theoretical variance/mean ratio of the neutron population in the subcritical condition of the AFRRI-TRIGA Reactor.

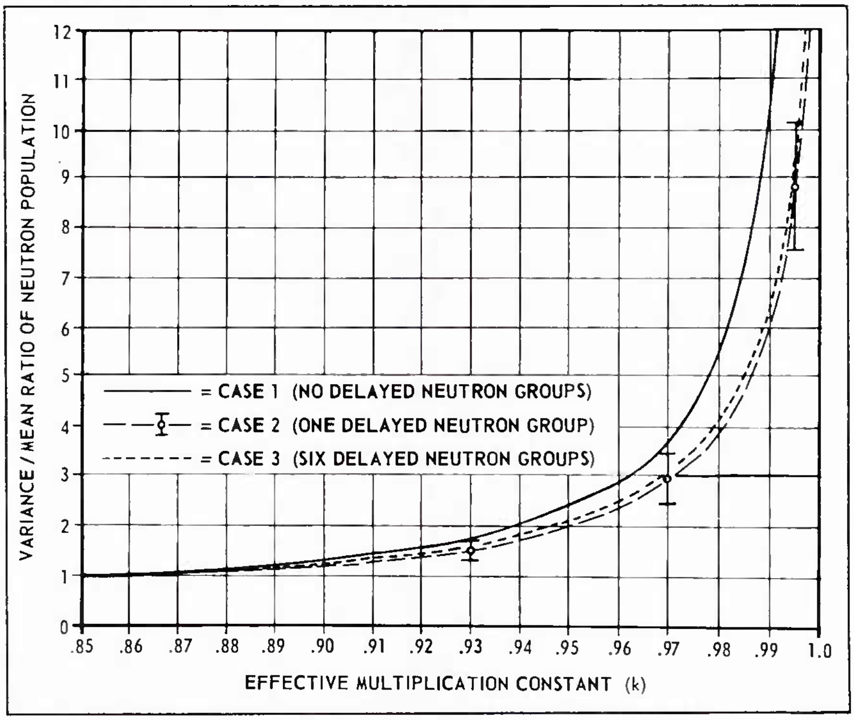

Figure 2. Theoretical variance/mean ratio of the neutron population in the subcritical condition of the AFRRI-TRIGA Reactor.

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The numerical evaluation of the theoretical variance/mean ratio of the neutron population in the subcritical condition of the AFRRI-TRIGA reactor is given for (1) the case of no delayed neutron groups; (2) the case of one equivalent delayed neutron group; and (3) the case of six delayed neutron groups. The Courant-Wallace equations were solved to obtain the theoretical variance/mean ratio using the mean prompt neutron lifetime of 39×10^{-6} seconds in the AFRRI-TRIGA reactor.

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13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

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